Moderated mediation and J-curve emergence in path models: An information systems research perspective

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Abstract

Purpose. *J-curve relationship analyses can provide valuable insights to information systems (IS) researchers. We discuss moderated mediation in IS research and the related emergence of J-curve relationships.* ***Design/methodology/approach.*** *Building on an illustrative study in the field of IS, we lay out three steps to combine moderation and J-curve analyses, with the goal of more fully understanding the underlying moderated mediation relationships. We propose a new segmentation delta method to test for J-curve emergence, as part of this framework.* ***Findings.*** *We show, in the context of this study, the complementarity of moderation and J-curve analyses.* ***Research limitations/implications.*** *Currently, IS researchers rarely conduct moderation and J-curve analyses in a complementary way, even though there are software tools, and related methods, which allow them to do so in a relatively straightforward way.* ***Originality/value.*** *Our analyses were conducted with the software WarpPLS, a widely used tool that allows for moderated mediation and J-curve analyses, in a way that is fully compatible with the set of steps presented in this paper.*

Keywords: Path Analysis; Moderation; Nonlinearity; J-Curve; Information Systems; Internet Use; Government Corruption

Introduction

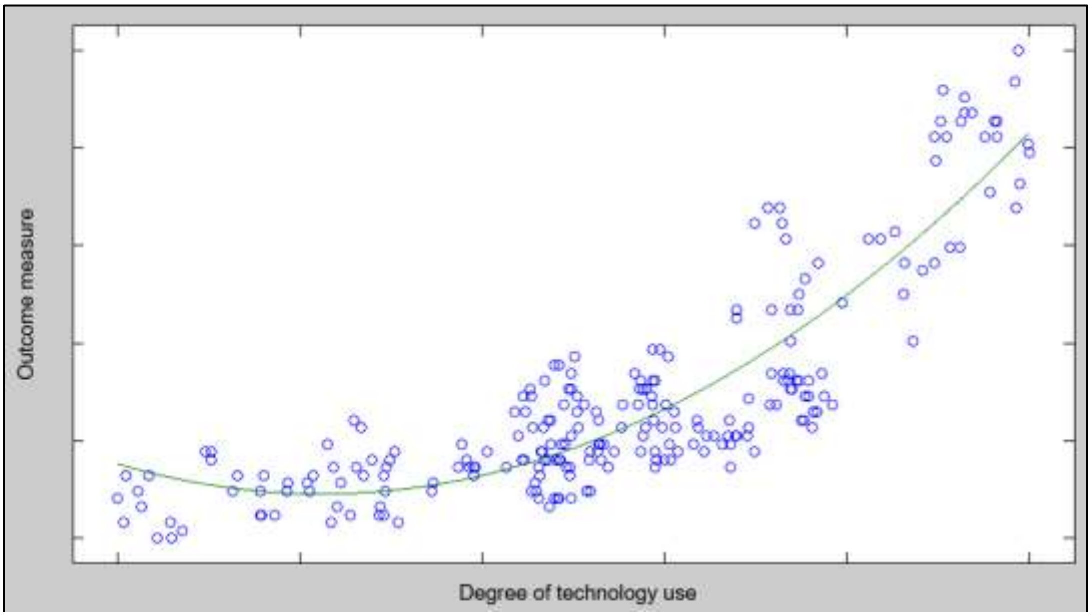
Information systems (IS) research is often concerned with the effects that information and communication technologies have on individuals and groups when they are adopted to carry out intermediate tasks, such as project management; which will produce main outcomes of interest, such as enhanced job performance (Elie-Dit-Cosaque & Straub, 2011; Kettinger & Yi, 2010; Kock, 2017; Kock & Moqbel, 2016; Kock et al., 2006; Meske et al., 2019). Typically these technologies act as facilitators in the performance of the tasks, without which the desirable outcomes are not obtained. In this type of scenario, the task performance facilitation effect is a mediating effect (Henttonen & Blomqvist, 2005; Kettinger & Yi, 2010; Kock & Moqbel, 2016).

Since it takes a significant amount of acquired competence to use technologies as facilitators in the performance of non-trivial tasks, low levels of technology use may lead to low or even negative results in terms of desirable outcomes (DeSanctis & Poole, 1994; Kock et al., 2006). As individuals and groups become more effective at using technology, greater use ensues; which is likely to make the task facilitation effect on desirable outcomes both positive and progressively stronger (DeRosa et al., 2004; Standaert et al., 2016). This characterizes a moderated mediation effect, which tends to lead to the emergence of a J-curve relationship with the general shape indicated in Figure 1.

J-curve relationships present patterns that are challenging to analyze through data segmentation; e.g., by segmenting a dataset into, say, 3 segments and analyzing each segment separately through linear methods. The reason for this is that at the extremes of the J-curve frequently the gradient of standardized variation (i.e., the "local" path coefficient, which varies along the curve) will be greater than 1. In this paper we provide an example where gradients of close to 2 are observed. Normally a path coefficient greater than 1 will not be obtained in a linear analysis without massive collinearity, which would likely distort many of the results of the analysis. In fact, in a linear analysis, even a path coefficient of 0.835 will be strong indication of pathological collinearity (Kock & Lynn, 2012).

In addition to the critical problem above, data segmentation also reduces the sample size available for the analysis of each segment, and thus the statistical power of each separate analysis. Moreover, data segmentation assumes that underlying heterogeneity is fragmented. This assumption is conceptually incompatible with the notion of moderation, which is expected to lead to "curvy" relationships, and thus with important theoretical considerations that may have led a researcher to hypothesize the existence of a moderating link in the causal model summarizing the theory being tested. For these and other related reasons, which will be illustrated in this paper, J-curve relationships should be analyzed through nonlinear methods.

Figure 1: J-curve emerging from moderated mediation



In the context of path models, moderated mediation and J-curve analyses lead to results that are often complementary but not redundant with one another. Because of this, we hope that our illustration will motivate IS researchers to strongly consider conducting both types of analyses. Currently, this is rarely the case, even though there are software tools and related methods that allow IS researches to conduct both types of analyses relatively easily. Normally moderation analyses are conducted in IS research, but not J-curve analyses.

The focus here is on path models, where variables that are relevant in the context of IS research are causally linked, with the direction of the causal relationships being hypothesized based on theory and past empirical research. To make our contribution more generic and accessible, we do not dwell on issues regarding indirect measurement with error. That is, in cases where multiple indirect measures are used for each variable, we assume that single variable scores also exist, which are the ones that we represent as variables in the discussion presented in this paper. In the context of structural equation modelling, these would be latent variable scores, which can be obtained regardless of the type of structural equation modelling method employed (Kock, 2019).

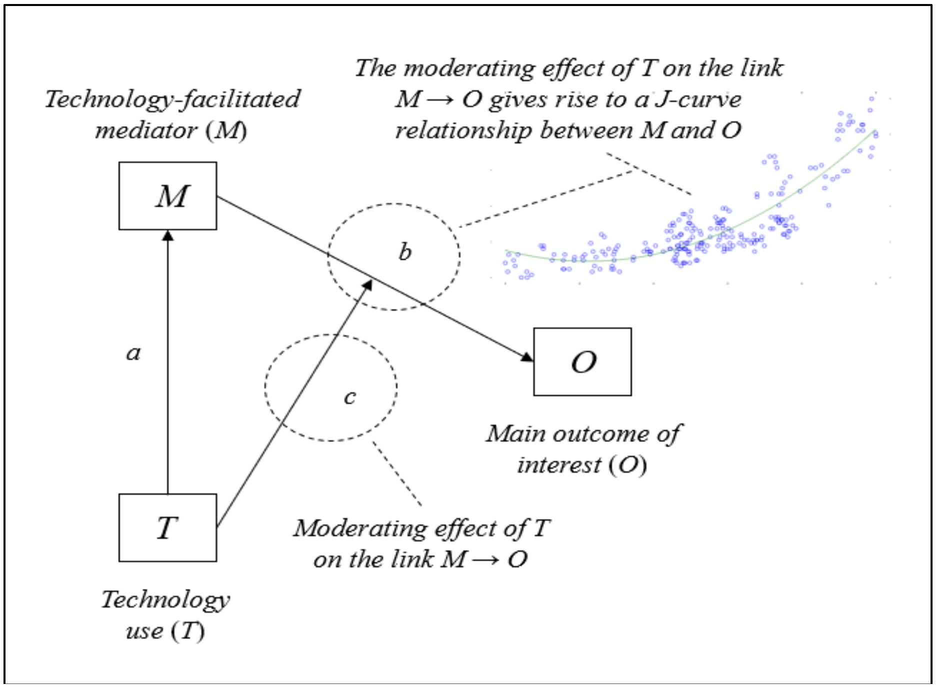
Our paper is organized as follows. We start by discussing moderated mediation in IS research and the related emergence of J-curve relationships. Next we introduce an illustrative study in the field of IS, and proceed to lay out three steps to combine moderation and J-curve analyses targeting moderated mediation relationships. We show, in the context of this study, the complementarity of moderation and J-curve analyses, which we highlight in our discussion. To make our exposition more straightforward, and without any impact on the generality of our discussion, we assume that all variables are standardized unless stated otherwise – i.e., scaled to have a mean of zero and standard deviation of one.

Moderated mediation in IS research

Information and communication technologies are usually adopted by individuals and groups so that they can carry out tasks that will produce main outcomes of interest. In this sense, a technology typically facilitates the performance of a task, without which desirable outcomes are not likely to be obtained. For example, the Internet can be used by media organizations to provide information to the public about government decisions, and thus increase the "transparency" of the government decisions. Increased transparency is likely to be associated with decreased government corruption, and thus with increased government honesty – the opposite of corruption, and the main outcome of interest in this example.

The degree to which the public is informed in an accurate way about government decisions in a country can be measured as the "transparency" of the country, for which scores are published by the World Bank through its Voice and Accountability Index (Globerman & Shapiro, 2003; Kock & Gaskins, 2014). Without an effort to achieve high transparency by media organizations, Internet use may not, at least not by itself, foster government honesty. Moreover, since it takes a significant amount of acquired competence for media organizations to effectively use the Internet to increase government honesty via transparency, at low levels of Internet use the effect of transparency on government honesty may be low or even negative. As media organizations become more effective at using the Internet, greater use is likely to make the effect of transparency on honesty both positive and progressively stronger. A generic representation of this scenario is provided in Figure 2.

Figure 2: Moderated mediation model



Here the variable *T* (technology use) causes *M* (technology-facilitated mediator), which in turn causes *O* (the main outcome of interest). The variable *T*, in addition to causing *M*, also moderates the link *M* → *O*. In our example above, *T* is Internet use, *M* is transparency, and *O* is government honesty. That is, increased Internet use is predicted to be associated with increased transparency, which is in turn predicted to be associated with increased government honesty. Moreover, increased Internet use is predicted to be associated with a positive growth in the slope of the association between transparency and government honesty. As we explain in more detail in Appendix A, this often gives rise to a J-curve relationship between *M* and *O*.

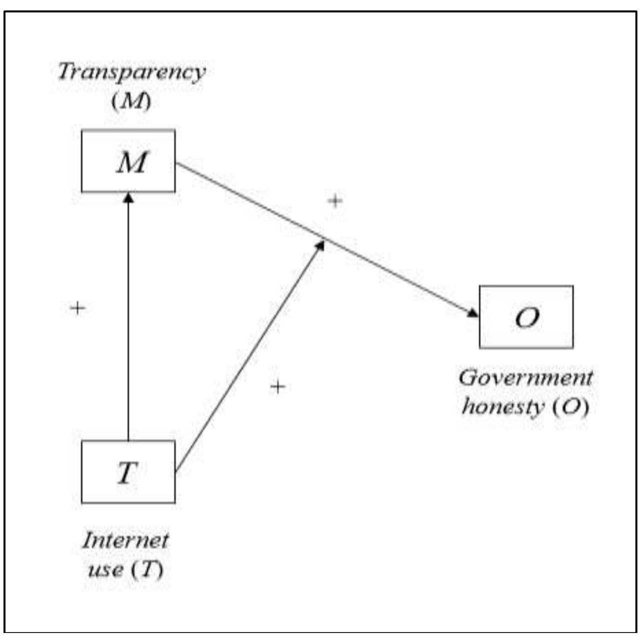
For simplicity we assume that *M* fully mediates the relationship between *T* and *O*. That is, we assume that the path coefficient for the direct link *T* → *O* is zero, which is the same as deeming the link to be nonexistent. This does not necessarily have to be the case, as there may be other mediators or a significant direct effect of *T* on *O* in addition to the indirect effect. The assumption that *M* fully mediates the relationship between *T* and *O*, adopted here, does not detract in any substantial way from the generality of our discussion.

This generic model incorporates our belief that moderated mediation is often present in IS phenomena, because the successful use of technology to accomplish outcomes takes competence that must be acquired through use itself. That is, we expect technology use *T* by individuals or groups to not only influence a mediator (*M*) directly, but to also moderate the relationship between the mediator *M* and the main outcome of interest *O*. The more the technology is used to increase the mediator, the more competence is acquired in its mediated use, and thus the stronger is the effect of the mediator on the main outcome of interest.

Illustrative study

This section briefly introduces an illustrative study in the field of IS. The study examines relationships among the variables Internet use (*T*), transparency (*M*), and government honesty (*O*). We employ the generic model presented above, using the same variable symbols (see Figure 3). This study is used here only for illustration purposes, and is not meant to be an empirical contribution. The data used covers 47 developing countries, 24 in Latin America and 23 in Sub-Saharan Africa; and spans 5 years (2006 to 2010), adding up to a total sample size of 235 (47 x 5). Developing countries tend to present a wide variation in Internet use, which is helpful for our illustration purposes.

Figure 3: Illustrative study



The measures used are adapted from those employed by Kock & Gaskins (2014). Internet use was measured by the number of Internet users per 100 inhabitants in a country, obtained from the

World Bank. Transparency was measured through the Voice and Accountability Index, also from the World Bank. Government honesty was measured through the Corruption Perceptions Index published by Transparency International. To facilitate interpretation of the results, both transparency and government honesty were obtained by standardization of the original indices. Since the data spans multiple years, we conducted a full latent growth analysis (Kock, 2020) to check whether any of the direct links in the model experienced growth, whether negative or positive, associated with the year in which the data was collected. As will be discussed in more detail later, our results suggested no growth, which means that the results of the moderation and J-curve analyses presented in the following sections hold for each of the years in which data was collected.

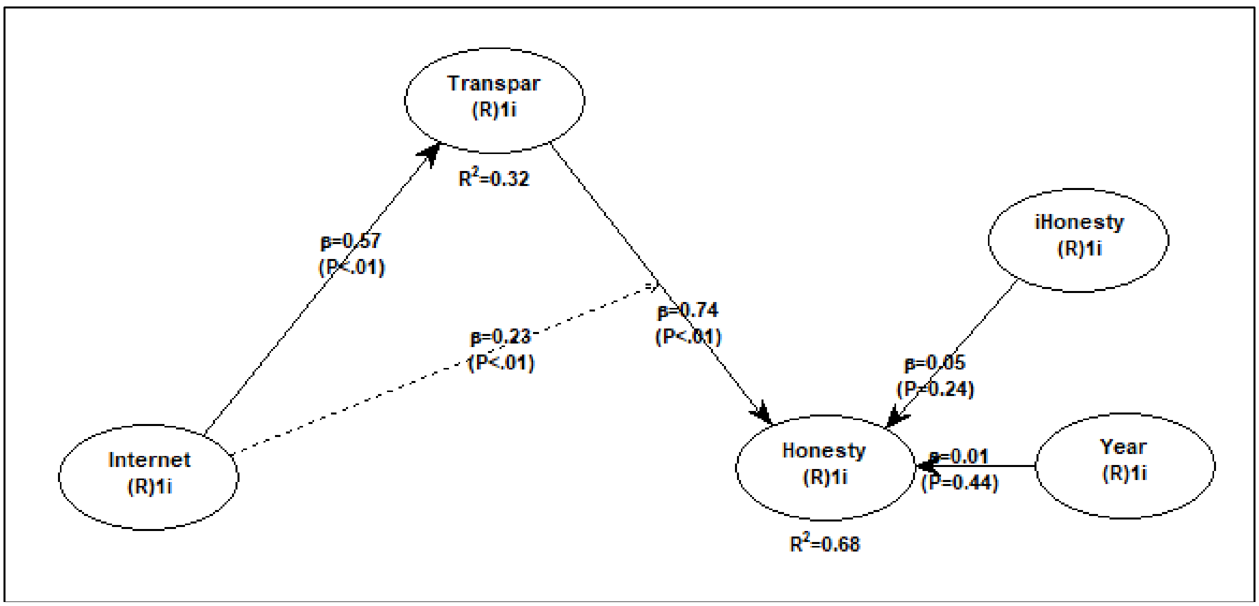
The study builds on modernization theory, a broad theory of social change that attempts to identify those factors that lead to positive and negative social change, among which technological diffusion is predicted to play a key role (Apter, 1965; Corrales & Westhoff, 2006; Cooks & Isgro, 2005; Scott, 1995). While neither the theory nor the empirical studies that led to it or validated it are the foci of this paper, the fact that our illustrative study is based on a carefully developed and tested theory lends credence to the viability of the model underlying the study. In our methodological exposition this is regarded as the true population model in the context of the illustrative study.

Our analyses were conducted with the software WarpPLS 6.0 (Kock, 2018), because this widely used software conveniently allows for moderated mediation and J-curve analyses (Guo et al., 2011; Schmiedel et al., 2014; Schmitz et al., 2016; Wilson & Djamasbi, 2019), in a way that is fully compatible with the set of steps presented in this paper. The free trial version of this software is a full implementation (not a demo version) and is available for approximately 3 months. Moreover, this software estimates all of the coefficients needed for a full illustration of the phenomena targeted here. The software features employed yielded intermediate and final results that were checked with other widely used software tools such as Excel, SPSS, MATLAB, and various R packages. These various checks involved extensive manual work, and generally suggested that the features yield trustworthy results (see Appendix B).

Step 1: Moderation analysis

Figure 4 summarizes the results of the moderation analysis conducted with WarpPLS, in the context of moderated mediation in our illustrative study. The analysis focuses on the moderation of Internet use (*T*) applied to the mediated relationship between transparency (*M*) and government honesty (*O*). The path coefficients for the direct relationships $T \rightarrow M$ and $M \rightarrow O$ were found to be both significant: respectively .57, P < .01; and *b* = .74, P < .01. The path coefficient associated with the moderating effect was also found to be significant: .23, P < .01. In this analysis, all paths are set as linear in WarpPLS. This includes the path associated with the moderating effect.

Figure 4: Moderation analysis results



Notes: Internet = Internet use (*T*); Transpar = transparency (*M*); Honesty = government honesty (*O*); iHonesty = instrumental variable (*iT*) accounting for variation from *T* that ends up in *O*; Year = year (Y).

Internet use (*T*) is hypothesized in our illustrative study to have no direct effect on government honesty (*O*), or a corresponding path coefficient of zero for that direct effect. That is, Internet use (*T*) is hypothesized to influence government honesty (*O*) only indirectly. However, since there is variation that flows from *T* to *O* via transparency (*M*), this creates the potential for endogeneity (Wooldridge, 2015) with respect to *O*, which could have distorted the path coefficients.

This was addressed through the inclusion of an instrumental variable iHonesty (*iT*) that incorporates the variation from *T* that ends up in *O* and nothing else (Kock, 2018). The menu option “Explore analytic composites and instrumental variables” was used for this in WarpPLS, with the sub-option “Single stochastic variation sharing”. The underlying technique, variation sharing, is discussed by Kock & Sexton (2017). The path coefficient for the link $iT \rightarrow O$ is small, at .05, and nonsignificant. As an implementation of the Heckman procedure for endogeneity assessment and control (Bascle, 2008; Certo et al., 2016), this suggests that there is no significant endogeneity with respect to O in our model.

The data in our illustrative study was collected over multiple years, which opens the door for the year in which the data has been collected to influence government honesty (O) in a way that may compete with the direct effect of transparency (M) or the moderating effect of Internet use (T) on the link $M \rightarrow O$. This could have also distorted the path coefficients, and was addressed through the creation of a variable storing the year in which each data point was collected; shown as year (Y) in the model, and pointing at government honesty (O). The path coefficient for the link $Y \rightarrow O$ is small, at .01, and nonsignificant; suggesting that year (Y) has no significant direct influence on government honesty (O) in our model. This link also allows us to control for the $Y \rightarrow O$ effect, allowing us to conclude that the significant effects represented by the path coefficients occur regardless of the year in which data was collected.

In addition to controlling for year (Y) in the model, we also checked whether any latent growth (Kock, 2020; Singer & Willett, 2003) was associated with year (Y). We did so by investigating whether any of the path coefficients in the model was significantly influenced by year (Y), employing the “full latent growth” feature of WarpPLS (Kock, 2018; 2020). The results suggested no significant variation in any of the path coefficients in the model in response to variations in year (Y).

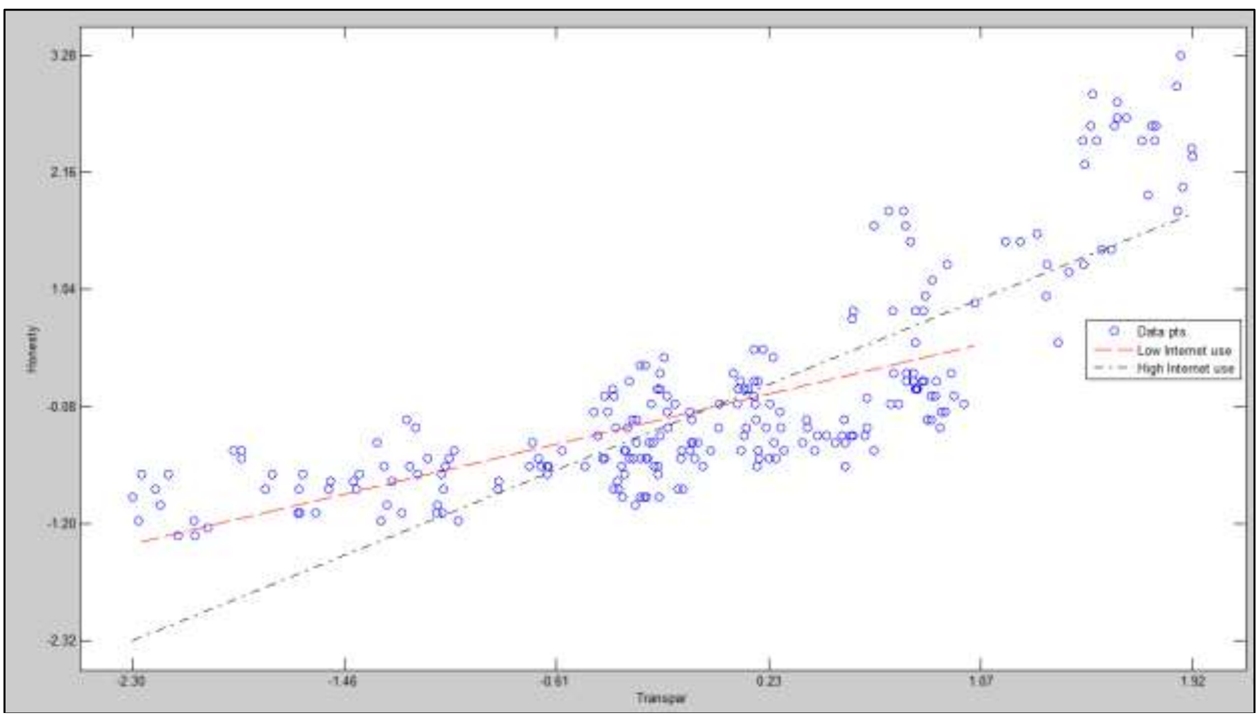
Moderation analyses typically require the inclusion of a new interaction effect variable to the model for the estimation of each coefficient associated with a moderation effect. This can add vertical or lateral collinearity to the model (Carte & Russel, 2003; Kock & Lynn, 2012), to the point of distorting path coefficients. The calculation of full collinearity variance inflation factors (VIFs) and their comparison against the threshold of 5 has been proposed to assess whether pathological collinearity exists in a path model (Kock & Lynn, 2012). Table 1 shows these full collinearity VIFs for our model, obtained through the “View latent variable coefficients” menu option of WarpPLS. As we can see, none of them is above the threshold of 5.

Table 1: Full collinearity VIFs

T	*M*	*O*	*Y*	*iT*	*T*M*
2.667	3.944	3.331	1.120	1.342	1.676

Normally moderation effects are graphically illustrated through: (a) segmentation of the dataset into high and low values of the moderating variable; and (b) estimation and plotting or the best-fitting regression lines for each data segment. We did this and show the results in Figure 5, using the "View moderating relationship in one graph with data points" menu option of WarpPLS. Note that for low values of Internet use (*T*) the slope of the best-fitting regression line is positive and of lower magnitude than the corresponding slope for high values of Internet use (*T*), which is also positive. This provides a graphical illustration of the positive path coefficient associated with the moderating effect, .23, in our model. That is, as Internet use (*T*) increases, from low to high, so does the positive hypothesized effect of transparency (*M*) on government honesty (*O*).

Figure 5: Moderation relationship graph



As we have pointed out earlier, moderated mediation effects such as the one in our model tend to give rise to J-curve relationships. As it will be seen later, analyses targeting such J-curve relationships provide insights that are complementary to, and difficult to obtain from, moderation analyses alone. However, we must first test whether moderated mediation has indeed given rise to a J-curve relationship whose curvature is “significant” – i.e., “curved” enough to be profitably targeted in a complementary J-curve analysis. This is done through a J-curve emergence test.

Step 2: J-curve emergence test

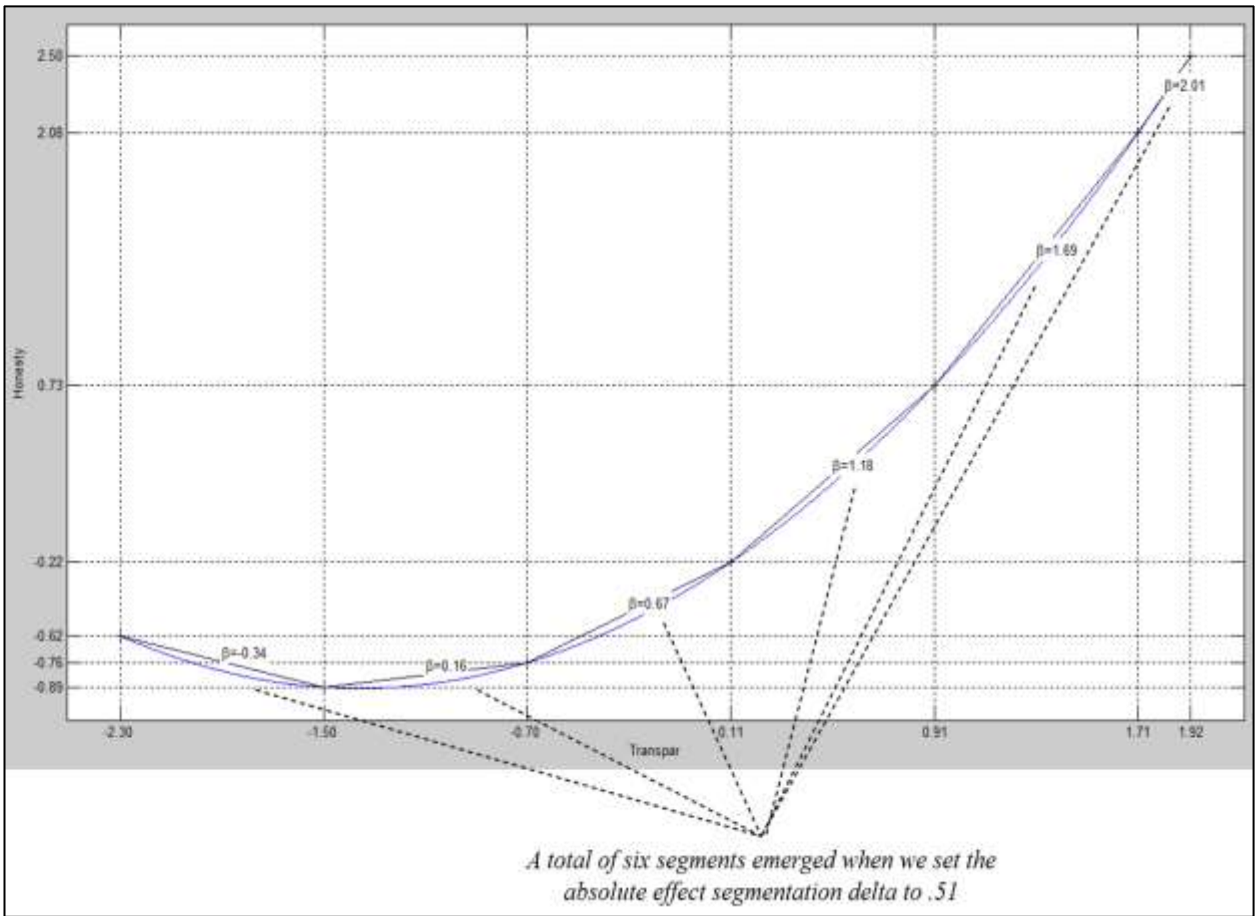
Let us consider Cohen’s (1988; 1992) power assessment guidelines in the context of a simple model with only one predictor and one criterion latent variable. A path coefficient whose value that would satisfy $\beta^2/(1-\beta^2) > .35$ in this model, which is used to calculate Cohen’s f^2 effect size coefficient, would be associated with a large effect size. Such a path coefficient would be .51 or higher.

The threshold value of .51 could be used as a basis for a method for J-curve identification, based on what we refer to here as the “segmentation delta method”. In this method we would try to segment a nonlinear graph so that successive estimates of the first derivatives of the underlying nonlinear function would have gradients that would be .51 greater than the previous gradients. The first derivatives define lines that are tangents to the nonlinear function. If we can obtain three segments or more, this is an indication that an actual J-curve emerged from the moderated mediation. Thus we can conclude that we should conduct a full J-curve analysis to supplement a moderation analysis. We can show how this J-curve identification test would work based on our illustrative study (see Figure 6).

Figure 6 shows the results of a data segmentation analysis using WarpPLS where the “View focused multivariate relationship graph with segments (standardized scales)” menu option is used for the relationship between transparency (*M*) on government honesty (*O*), which is set as a “Warp2” relationship in WarpPLS (a J-curve shape relationship). In this option the “absolute effect segmentation delta” chosen through the “Settings” menu option is .51. As we can see, a total of six segments emerged from the analysis, which is more than the minimum of three segments needed, thus we can conclude that an actual J-curve emerged from the moderated mediation based on the segmentation delta method.

The segmentation delta method for J-curve emergence testing discussed above focuses on testing whether moderated mediation gives rise to a J-curve relationship whose curvature is large enough to be targeted in a complementary J-curve analysis. This complementary analysis is discussed in the section below.

Figure 6: Segmentation delta method



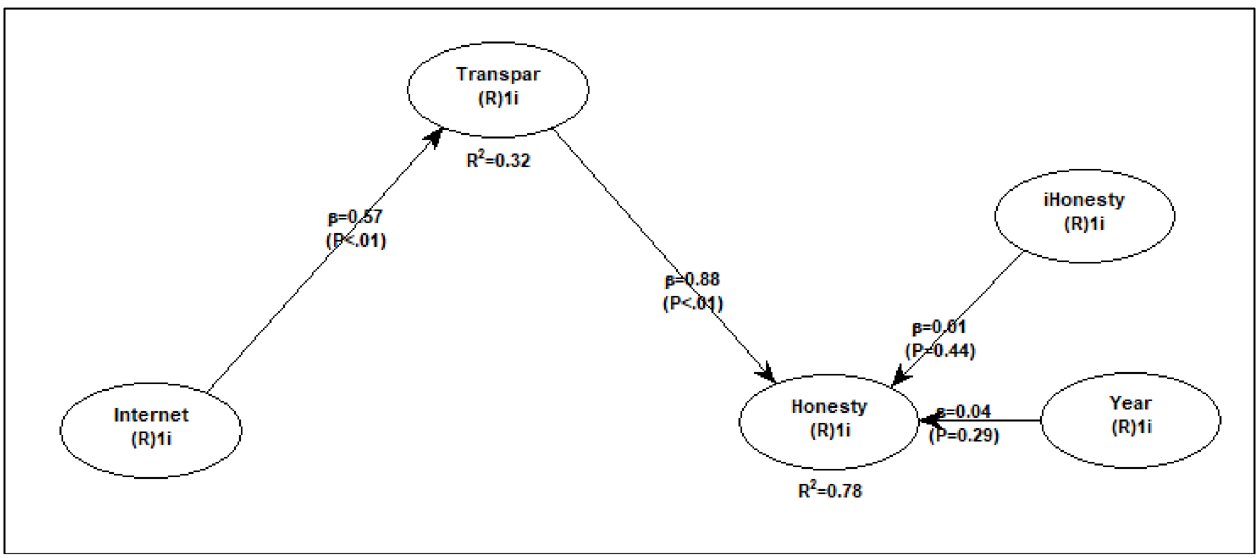
Notes: each coefficient shown is an estimate of the curve's gradient for the segment; since 3 segments or more emerged = an actual J-curve emerged.

An alternative method for J-curve emergence testing, building on what is often referred to as the Satterthwaite method (Kock, 2014), is presented in Appendix C. This alternative method could be used in addition to the segmentation delta method in cases where researchers are asked (e.g., by a paper review panel) to test whether path coefficients from linear and nonlinear analyses are significantly different.

Step 3: J-curve analysis

Figure 7 presents the results of a J-curve analysis where the causal link $M \rightarrow O$, between transparency (M) and government honesty (O), is hypothesized to have a J-curve shape, as this was suggested by the J-curve emergence test. The path coefficient associated with this link was obtained through the solution of the equation: $O = hF(M) + \varepsilon_4$; whereby the link was modeled as a "Warp2" relationship in WarpPLS, which is the type of relationship in that software that is associated with a J-curve shape. This path coefficient was found to be significant (h = .88, P < .01). The function $F(M)$ takes the standardized form $fM + gM^2$, and "warps" (i.e., linearizes) M prior to the calculation of the nonlinear coefficient of association h. This is essentially a linear regression after a nonlinear transformation. The function $F(M)$ can easily be obtained through a second-degree polynomial interpolation (see Appendix B), which is how we obtained it using WarpPLS.

Figure 7: J-curve analysis results



Notes: Internet = Internet use (T); Transpar = transparency (M); Honesty = government honesty (O); iHonesty = instrumental variable (iT) accounting for variation from T that ends up in O; Year = year (Y).

Note that this model used in the J-curve analysis does not incorporate the moderating effect, because the moderating effect captures some of the nonlinearity explicitly modeled in the J-curve analysis. That is, the moderation analysis and the J-curve analysis are "two facets of the same coin", and should be conducted with two different models: (a) the first with the moderating effect $T \rightarrow (M \rightarrow O)$ included, an all links modeled as linear; and (b) the second without the moderating effect, and with the link $M \rightarrow O$ modeled as nonlinear ("Warp2" relationship in WarpPLS).

The interpretation of the sign of the coefficient h is that it reflects the overall linear sign of the relationship; in other words, the sign of the corresponding path coefficient associated with the best-fitting line for the $M \rightarrow O$ relationship. The coefficient h itself can be interpreted as a measure of dispersion of the data points around the J-curve given by the function $F(M)$. The closer the points are to the J-curve, the higher is the path coefficient.

As with the moderation analysis, the calculation of full collinearity VIFs and their comparison against the threshold of 5 was also carried out here in this J-curve analysis, to assess whether pathological collinearity exists in this modified nonlinear path model (Kock & Lynn, 2012). Table 2 shows these full collinearity VIFs for our model. As we can see, none of them is above the threshold of 5. Moreover, when compared with those from the moderation analysis, these full collinearity VIFs are lower. The reason for this is that, since moderation is not explicitly analyzed here, no interaction variable had to be added to the model.

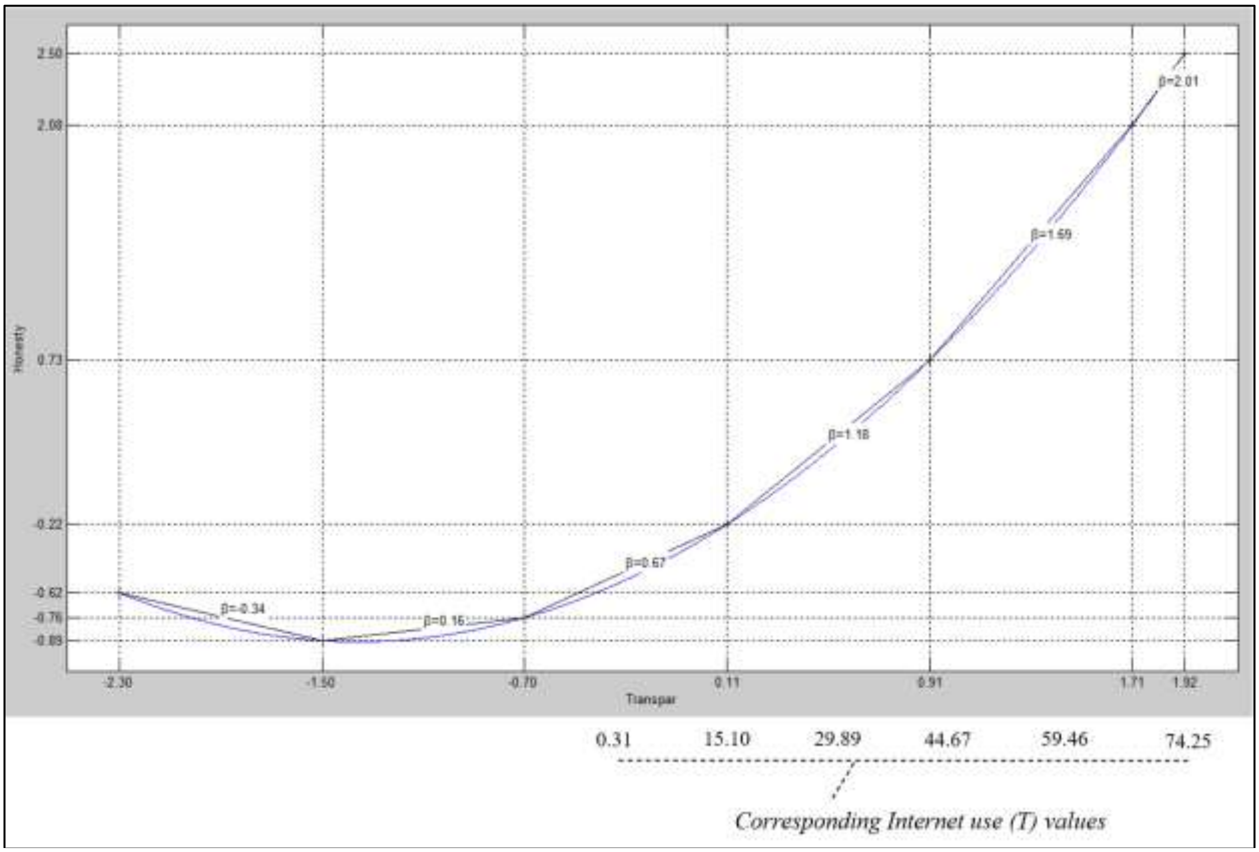
Table 2: Full collinearity VIFs

T	*M*	*O*	*Y*	*iT*
1.968	3.209	2.720	1.113	1.325

Figure 8 shows the segmented J-curve, as before, with various lines showing linear path coefficients and corresponding P values. It should be stressed that these linear path coefficients are not generated based on data segmentation, but rather on the segmentation of the best-fitting J-curve. At the bottom of the figure we see the Internet use (T) values associated with various transparency (M) values. These can be obtained with the WarpPLS menu option "View focused multivariate relationship graph with segments (unstandardized scales)" for the $T \rightarrow M$ relationship.

As we can see, the value of Internet use (T) has to be a above a certain threshold, of approximately 45 users per 100 people, for us to expect transparency (M) to be high. Since transparency (M) is a standardized variable, with an average of 0 (zero), the value of 1 would be associated with high transparency (M). Above this point, the relationship $M \rightarrow O$ between transparency and government honesty becomes very strong, with path coefficients between 1.69 and 2.01. That is, one standard deviation increase in transparency (M) causes an increase of between 1.69 and 2.01 standard deviations in government honesty (*O*). It is unlikely that effects of this magnitude would be uncovered by a linear analysis with data segmentation, because typically path coefficients greater than 1 can only happen in linear analyses due to distortions stemming from massive multicollinearity.

Figure 8: J-curve graph segments



Based on the above discussion, we can conclude that a J-curve analysis provides research insights that are not available from a moderation analysis. This occurs even though the existence of moderated mediation is fundamentally what leads to the emergence of the corresponding J-curve relationship. Therefore, we recommend that researches consider conducting J-curve analyses in conjunction with moderation analyses, presenting the results of the two types of analyses as complementary.

Discussion

In cases where moderated mediation occurs, we recommend that researchers perform the analysis in three main steps. Step 1 entails conducting a moderation analysis. Step 2 entails a J-curve emergence test, to ascertain whether moderated mediation gives rise to a J-curve relationship. If Step 2 suggests the existence of a clear J-curve relationship, then researchers should proceed to a J-curve analysis, whose results should be presented as complementary to the results of the moderation analysis.

The results of the moderation and J-curve analyses provide complementary insights into what are clearly “two facets of the same coin”. While it is moderated mediation that gives rise to a J-curve relationship, the shape of the relationship can only be grossly modeled through segmentation of the dataset into high and low values of the moderating variable and estimation and plotting or the best-fitting regression lines for each data segment. Both, moderation and J-curve analyses are needed for a full understanding of the underlying patters arising from moderated mediation.

In some cases, moderation analyses will not be feasible, as suggested by the different full collinearity VIFs obtained for the moderation and J-curve analyses. If T (technology use) and M (technology-facilitated mediator) are so highly correlated as to lead to at least one full collinearity VIF greater than 5, only a J-curve analysis may be possible without significant distortion of path coefficients due to multicollinearity. In these cases, the variables T and M may in fact be measuring the same underlying construct, leading to a situation where T actually moderates itself. This would be a case of self-moderation, which would also lead to the emergence of a J-curve.

The reader may at this point ask whether we should have employed an analysis based on a polynomial regression (Shacham & Brauner, 1997), by solving the equation below directly, instead of employing a intermediate transformation via a function $F(M)$ obtained through a polynomial interpolation. Arguably this approach should be avoided.

$$O = kM + lM^2 + \varepsilon_5.$$

The main reason why one should avoid this type of polynomial regression analysis is that in it the terms kM and lM^2 are treated as being derived from different variables, when in fact they are derived from the same variable M. The equation solved is essentially the one below, where $X = M$ and $Y = M^2$. Given that terms kM and lM^2 are derived from the same variable, these terms will share a certain amount of variation. In fact, prior to standardization, these terms will often tend to be pathologically colinear. The common variation may lead to distortions in the corresponding path coefficients.

$$O = kX + lY + \varepsilon_5.$$

Moreover, when we employ a polynomial regression analysis it becomes difficult to interpret the coefficients k and l, as they refer to components of the same nonlinear relationship. This interpretational difficulty persists even when the coefficients are not distorted by the presence of pathological collinearity.

An important point that technical readers may make is that the model assumed to have led to the emergence of a J-curve relationship is a causal model, with causality being at the source of the J-curve emergence. So, how can we assess whether the model is sound in terms of causality? WarpPLS provides three global model fit and quality indices that are aimed at causality assessment (Kock, 2018): Simpson's paradox ratio (SPR), R-squared contribution ratio (RSCR), and statistical suppression ratio (SSR). Acceptable values for each of these indices, listed below, have been proposed based on Monte Carlo simulations (Kock, 2018). As it will be seen below, these indices can be easily calculated manually.

The SPR index is a measure of the extent to which a model is free from Simpson's paradox instances, which are seen as indications of incorrect causal assumptions being incorporated into a model (Kock, 2015; Pearl, 2009; Wagner, 1982). In fact, a Simpson's paradox instance may be an indication that a model is missing a moderating link (unlike our illustrative model), and that there is a "hidden" J-curve relationship in the model (Kock & Gaskins, 2016). An instance of Simpson's paradox occurs when a path coefficient and a correlation associated with a pair of linked variables have different signs. The SPR index is calculated by dividing the number of paths in a model that are not associated with Simpson's paradox instances by the total number of paths in the model. It is proposed that acceptable values of SPR are equal to or greater than 0.7, meaning that at least 70 percent of the paths in a model are free from Simpson's paradox.

The RSCR index is a measure of the extent to which a model is free from negative R-squared contributions, which occur together with Simpson's paradox instances, and are thus seen also as indications of incorrect causal assumptions existing in a model (Kock, 2015; Pearl, 2009; Wagner, 1982). When a predictor latent variable makes a negative contribution to the R-squared of a criterion latent variable, this means that the predictor is actually reducing the percentage of variance explained in the criterion. It is proposed that acceptable values of RSCR are equal to or greater than 0.9, meaning that the sum of positive R-squared contributions in a model makes up at least 90 percent of the total sum of the absolute R-squared contributions in the model.

The SSR index is a measure of the extent to which a model is free from statistical suppression instances, also seen as indications of incorrect causal assumptions existing in a model (Kock, 2015; MacKinnon et al., 2000; Pearl, 2009). An instance of statistical suppression occurs when a path coefficient is greater, in absolute terms, than the corresponding correlation associated with a pair of linked variables. Normally one would expect the opposite to occur when models are correctly specified in terms of the directions of its various causal links: a path coefficient being of the same or lower magnitude than its corresponding correlation. Acceptable values of SSR are proposed to be equal to or greater than 0.7, meaning that at least 70 percent of the paths in a model are free from statistical suppression.

Table 3 shows the values of each of these three causality assessment indices for our path model. As we can see, our model is completely free from Simpson's paradox instances, negative R-squared contributions, and statistical suppression instances. So it appears that our model is largely free from indications of incorrect causal assumptions being incorporated into it; that is, the model appears to be generally sound in terms of causality.

Table 3: Values of causality assessment indices

Index	*Value*	*Assessment*
Simpson's paradox ratio (SPR)	1.000	acceptable if >= 0.7
R-squared contribution ratio (RSCR)	1.000	acceptable if >= 0.9
Statistical suppression ratio (SSR)	1.000	acceptable if >= 0.7

It is important to note that the SPR, RSCR, and SSR indices are meant to be used in conjunction with theory. In other words, the path model and its causal links must be based on sound theory, which would then be supported or not by the causality assessment indices. Having acceptable indices is not meant to be a decisive "proof" that the causal network of effects depicted by the model is correct. It simply means that the causal network of effects implied by the model finds empirical support in the context of the dataset used to empirically test the model.

Conclusion

We put forth evidence in the foregoing sections suggesting that moderated mediation in path models may lead to the emergence of J-curve relationships. We also showed that moderation and J-curve analyses lead to results that are often complementary, but not redundant with one another. Because of this complementarity, we argued that IS researchers should strive to conduct both types of analyses, particularly in those cases where the existence of a well-defined J-curve is clearly established. Currently, IS researchers rarely conduct moderation and J-curve analyses in a complementary way, even though there are software tools and related methods that allow them to do so. We provide a set of steps to guide researchers interested in doing these analysis.

Our arguments were made in the context of an illustrative study in the field of IS that examined relationships among the variables Internet use, transparency, and government honesty. The data in the illustrative study covered 47 developing countries, 24 in Latin America and 23 in Sub-Saharan Africa; and spanned 5 years. Since developing countries tend to present a wide variation in Internet use, the study was particularly helpful for our illustration purposes. Our analyses were conducted with the software WarpPLS 6.0, as this widely used software conveniently allows for moderated mediation and J-curve analyses.

While it is moderated mediation that gives rise to a J-curve relationship, the shape of the relationship can only be grossly modeled through segmentation of the dataset into high and low values of the moderating variable. This is what is usually done in classic moderation analyses, with estimation and plotting of the best-fitting regression lines for each of the two data segments. Our illustrative analyses strongly suggest that often both, moderation and J-curve analyses, are needed for a full understanding of the underlying patters arising from moderated mediation.

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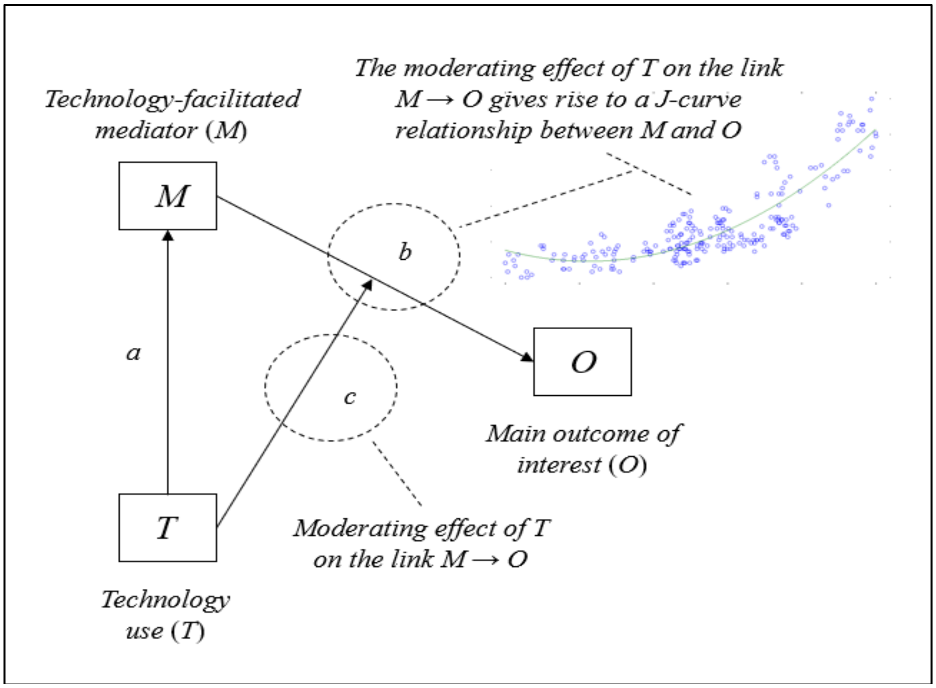
Appendix A: J-curve emergence

Since T causes M in our moderated mediation model (see Figure A.1), and thus M incorporates variation from T, a nonzero correlation between M and T arises.

Figure A.1: Moderated mediation model



With T being the only predictor of M in our model, this correlation consequently equals the path coefficient a. Thus we can conversely write T in terms of M as follows, where ε_1 is an uncorrelated error term that supplies the variation in T that is not shared with M.

$$T = aM + \varepsilon_1.$$

To simplify our discussion, we assume that M fully mediates the relationship between T and O (our main outcome of interest). As noted earlier, this does not necessarily have to be the case; e.g., there may be other mediators. With no impact on the generality of our discussion, we can write O in terms of M as follows, where ε_2 is an uncorrelated error term that accounts for the variance in O that is not explained by any of the other terms on the right size of the equation.

$$O = bM + cM(aM + \varepsilon_1) + \varepsilon_2.$$

The above equation can be rewritten as:

$$O = bM + acM^2 + cM\varepsilon_1 + \varepsilon_2.$$

Since both ε_1 and ε_2 are uncorrelated with M and any variable derived from it, the last two terms of the equation above can be replaced with ε_3. This is a single error term that is orthogonal to M and derived variables (e.g., M^2), and that incorporates the variation present in those other two error terms. We then have:

$$O = bM + acM^2 + \varepsilon_3.$$

Finally, we can rewrite the equation above, as shown below, explicitly referring to a function $F(M)$ of standardized form $fM + gM^2$ that "warps" (i.e., linearizes) M prior to the calculation of the nonlinear coefficient of association h. This function is obtained through a polynomial interpolation. In this new equation ε_4 accounts for the variance in O that is not explained by $F(M)$.

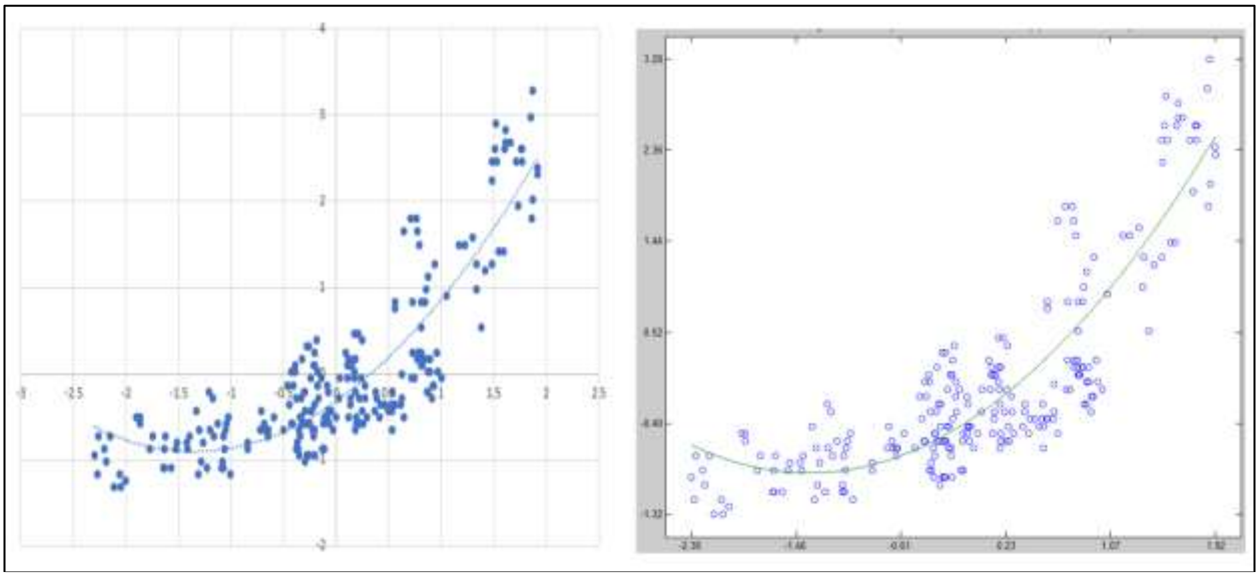
$$O = hF(M) + \varepsilon_4.$$

As we can see, the moderating effect of T on the link $M \rightarrow O$ gives rise to a quadratic relationship between O and M, represented through the function $F(M)$. This quadratic relationship, when plotted, will typically have the shape of a J-curve. Therefore, we refer to this type of relationship using the term "J-curve relationship".

Appendix B: Using Excel to obtain a J-curve

In order to model a relationship between two variables *X* and *Y* as a J-curve, WarpPLS first estimates the best-fitting quadratic function *F*, so that *Y* can be expressed as a function of *X* as *Y*=*F*(*X*). The software then regresses *Y* on *F*(*X*) and any other predictors. The regression coefficient for the *F*(*X*) term is thus different than the corresponding regression coefficient for *X*; if *Y* had been regressed on *X* instead of on *F*(*X*). The most critical element of this process is obtaining the best-fitting quadratic function *F*, which can be done with Microsoft Excel (see Figure B.1). Once that is done, the regression can be conducted with any of a number of software tools such as SPSS, MATLAB, and various R packages.

Figure B.1: Using Excel to obtain a J-curve



Notes: left = best-fitting quadratic function obtained with Excel; right = same function obtained with WarpPLS.

The left side of the figure shows the best-fitting quadratic function *F* obtained with Excel. The “Format Trendline” option in Excel was used, with the following settings selected: “Polynomial”, and “Order: 2”. The underlying equation is available through the option “Display Equation on chart”. The right side shows the same type of function obtained with WarpPLS. Here, all that was needed was to set the relationship as being of the “Warp2” type. The two functions are identical. The major advantage of using WarpPLS is that the software automates the entire process of J-curve modeling and corresponding nonlinear path analysis.

Appendix C: Satterthwaite method for J-curve emergence test

If we conduct a polynomial regression using the equation below, one could argue that the path coefficient associated with the link $M^2 \rightarrow O$, represented by *l* in the equation, would be a good candidate for J-curve identification. If *l* were to be statistically significant, then we should expect a clear J-curve pattern to have emerged.

$$O = kM + lM^2 + \varepsilon_5.$$

The main problem with this approach is that the terms kM and lM^2, treated as being derived from different variables, would often share so much variation (given that they are derived from the same variable *M*) as to be pathologically collinear. As a result, the coefficients *k* and *l* would be distorted. This would call into question a J-curve identification test building on *l* and its statistical significance.

An approach that we would offer as an alternative to the above is to calculate the ratio T_{jl} through the equation below, and then obtain the chance probability (i.e., the P value) associated with the ratio. In the equation, β_j is the path coefficient obtained for the link $M \rightarrow O$ through a J-curve analysis, β_l is the path coefficient obtained for the same link through a linear analysis, S_j is the standard error associated with β_j, and S_l is the standard error associated with β_l. This approach to testing whether two patch coefficients differ significantly from one another builds on what is often referred to as the Satterthwaite method (Kock, 2014).

$$T_{jl} = (\beta_j - \beta_l) / \sqrt{S_j{}^2 + S_l{}^2}.$$

The probability P_{jl} associated with the ratio T_{jl} can be obtained with a function such as *TDIST*, building on the incomplete beta function, as implemented in Microsoft Excel. This would be the probability that the ratio T_{jl} is associated with a chance event, and thus would normally be compared against the .05 threshold. The equation below shows how P_{jl} would be obtained, where: $ABS(T_{jl})$ is the absolute value of the ratio T_{jl}, and *N* is the sample size employed in the J-curve and linear analyses. The last argument in the *TDIST* function, the number "1", indicates that this is a one-tailed test because an absolute difference is being used in the test.

$$P_{jl} = TDIST\left[ABS(T_{jl}), (2N - 2), 1\right].$$

We can show how this J-curve identification test would work based on our illustrative study. From the study we know that the path coefficient from the J-curve analysis was .882 and the corresponding standard error was .056. The linear path coefficient from the moderation analysis was .742 and the corresponding standard error was .057. The sample size was 235. This yields a probability $P_{jl} = TDIST[ABS(1.752), 468{,}1] = .04$. Since this value is lower than .05, we can say that our J-curve identification test yielded a statistically significant probability that an actual J-curve emerged from the moderated mediation. Thus we can conclude that we should conduct a full J-curve analysis (as we did) to supplement a moderation analysis.